5.1 Introduction to Random Variables and Probability Distributions

______________________________ - any process by which an observation (or measurement) is obtained.

Examples:
1) Counting the number of eggs in a robin's nest.
2) Measuring the daily rainfall in inches.
3) Counting the number of defective light bulbs in a case of bulbs.
4) Measuring the weight in kilograms of a polar bear cub.

____

If x were to represent a quantitative variable that is measured in an experiment, we are then interested in the values that x will take on. X is a random variable because the value that x takes on in a given experiment is a chance or random outcome.

Two Types of Random Variables
1) Discrete Random Variables
2) Continuous Random Variables

____

When the observations of a quantitative random variable can take on only a finite number of values, or take on a countable number of values.

Examples of ______________________________________:
1) The number of students in a certain section of a statistics course this semester. This value must be a counting number such as 31 or 55. The values 23.12 and ½ are not possible.
2) The number of chicks living in a nest.
3) The number of students who vote in a given student body election.

____
When the observations of a quantitative random variable can take on any of the countless number of values in a line interval.

Examples of ____________________________________:
1) The air pressure in an automobile tire. The air pressure could in theory take on any value from 0 lb/in^2 (psi) to the bursting pressure of a tire. Values such as 20.126 psi, 20.121678 psi, and so forth are possible.
2) The heights of the students in this statistics class. The heights could in theory take on any value from a low of, say, 3 feet to a high of, say, 7 feet.
3) The number of miles per gallon fuel consumption of a car takes at random from the highway. In theory this could be any value from, say, 1 mile per gallon to 75 miles per gallon.

Probability Distribution:
A __________________________________ is an assignment of probabilities to the specific values of the random variable or to a range of values of the random variable.

A random variable has a probability distribution whether it is discrete or continuous.

The probability distribution of a discrete random variable has a probability assigned to each value of the random variable. The sum of these probabilities must be 1.

In a probability distribution, your probabilities column is calculated the same way we did relative frequency.

???	Frequency	Probability
	$\Sigma =$	$\Sigma =$

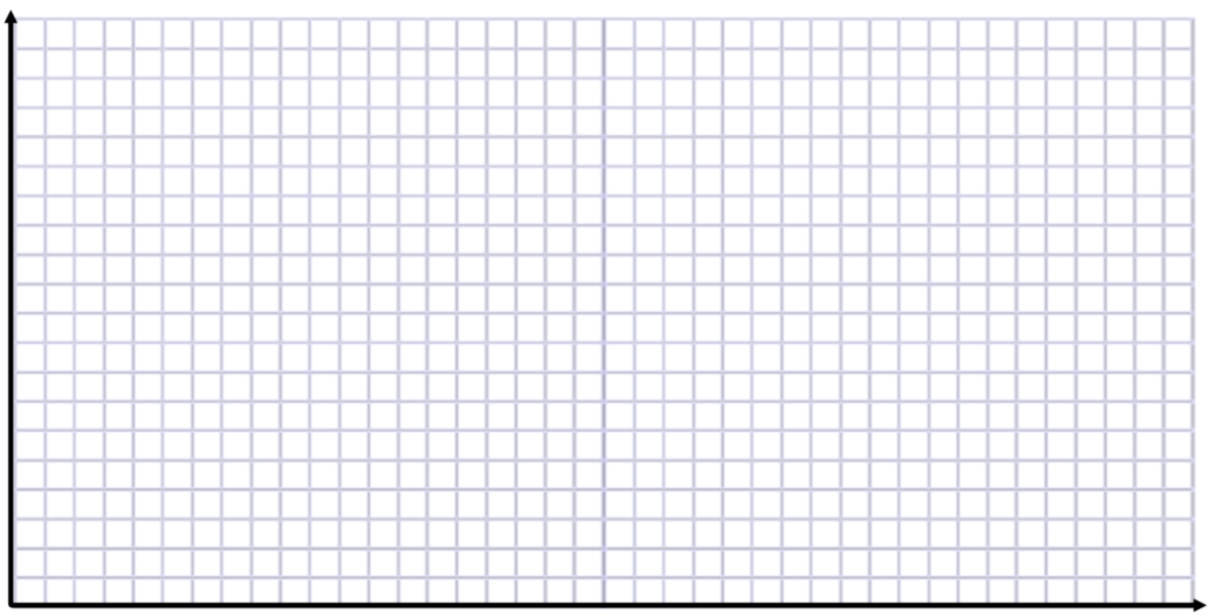
Let's look at an example:

A recent poll was taken of 125 participants. The poll asked on a scale of 1 – 6, rate McFadden's Restaurant, with 1 being the worst and 6 being the best. The table below gives the results of the poll:

Rating	**Frequency**	**Probability**
1	12	
2	11	
3	28	
4	18	
5	19	
6	37	
	$\Sigma =$	$\Sigma =$

To fill in the rest of the table, we just take each frequency for the given rating and divide it by the total number of participants.

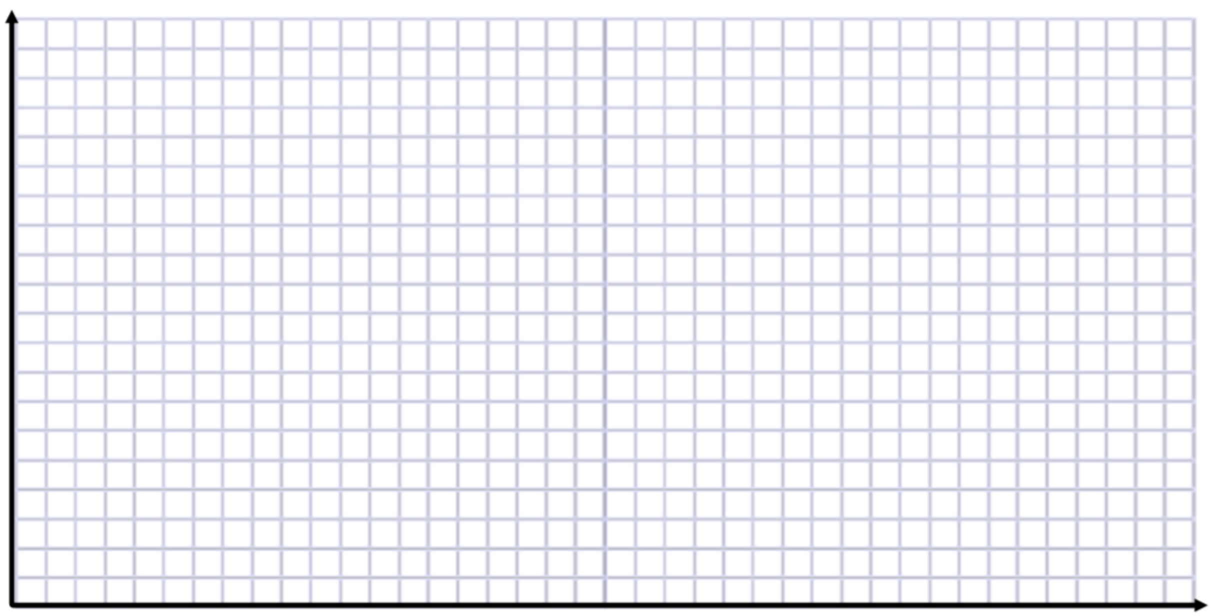
We can then graph the probability distribution as a histogram with probability on the vertical axis and rating on the horizontal axis.



Examples:

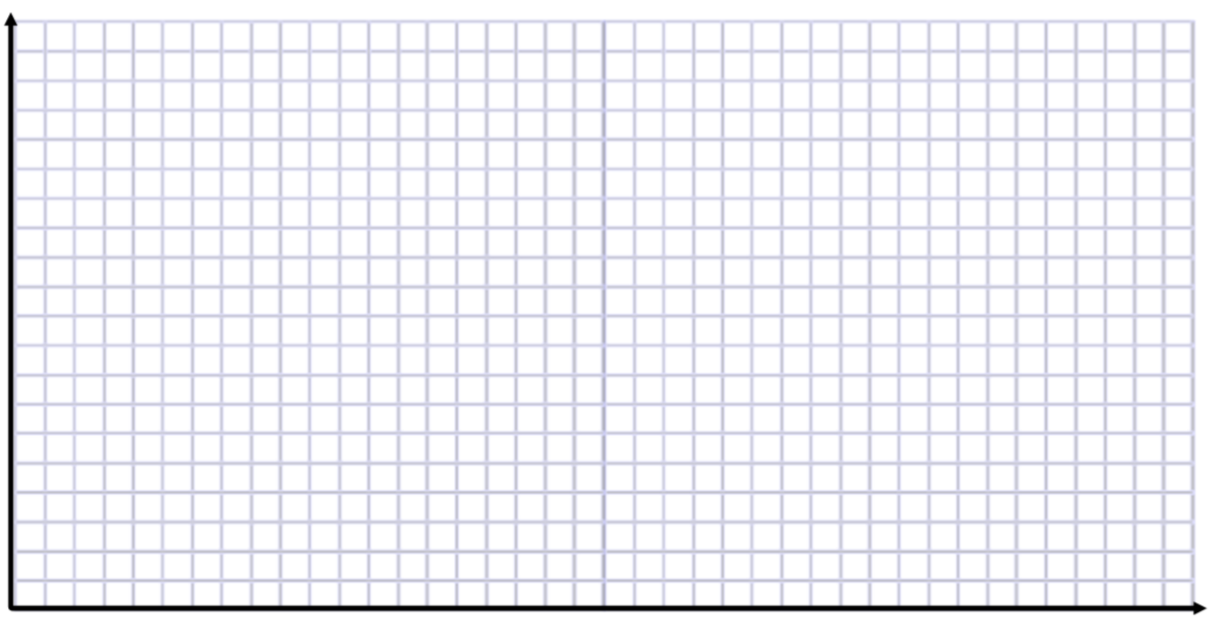
1) Complete the table below and graph the results:

	Frequency	**Probability**
1	65	
2	78	
3	111	
4	176	
	Σ =	**Σ =**



2) Complete the table below and graph the results:

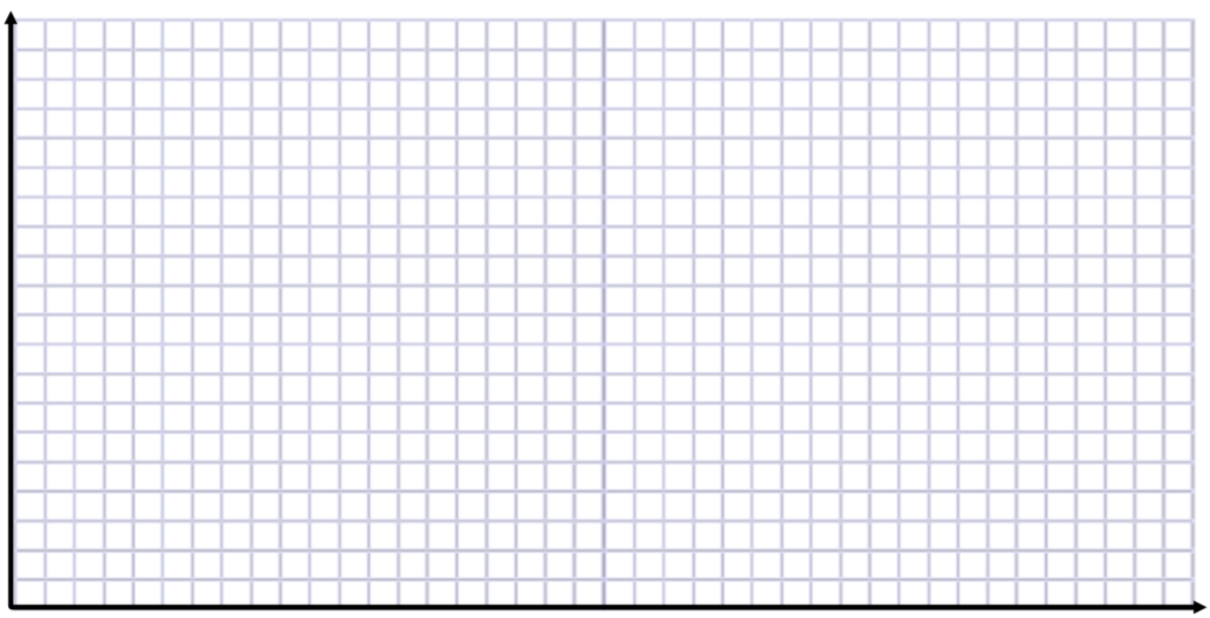
Rating	**Frequency**	**Probability**
0	245	
1	267	
2	222	
3	198	
4	210	
	Σ =	**Σ =**



3) A psychologist has devised a test that measures the congeniality factor (desire to work with people). A random sample of 1,000 adults took the test. Their scores are given in the following table:

x (Score)	Frequency of Adults with this Score
1	90
2	480
3	220
4	150
5	60

a) Construct a table to show the probability distribution.
b) Construct a histogram to display the data.



Mean & Standard Deviation of a Discrete Probability Distribution:
We have already discussed how a probability distribution can be thought of as a relative-frequency distribution. It has a mean and a standard deviation. If we are referring to the probability distribution of a _______________, then we use the Greek letter μ for the mean and σ for the standard deviation.

When you see those Greek letters used you immediately know the information given is for the ___________________________ rather than just a sample. These letters μ and σ are fixed numbers and are sometimes called parameters of the population.

The **mean** and the **standard deviation of a discrete population probability distribution** are found by using these formulas:

$\mu = \Sigma x P(x)$; μ is called the **expected value** of x

$\sigma = \sqrt{\Sigma (x - \mu)^2 P(x)}$; σ is called the standard deviation of x

where x is the value of a random variable,
$P(x)$ is the probability of that variable, and
the sum Σ is taken for all the values of the random variable.

Note: μ is the *population mean* and σ is the underlying *population standard deviation* because the sum Σ is taken over *all* values of the random variable (i.e., the entire sample space).

_______________________- mean of a probability distribution.
This term represents the idea that the mean is a "central point" or "cluster point" for the entire distribution. The mean or expected value is an average value, therefore, it need not be a point of the sample space.

1) Complete the following table and calculate the expected value and standard deviation:

x	**P(x)**	**xP(x)**	**x - μ**	**$(x - \mu)^2$**	**$(x - \mu)^2$ P(x)**
1	.12				
2	.27				
3	.12				
4	.34				
5	.15				
		Σ =			Σ =

2) Complete the following table and calculate the expected value and standard deviation:

x	**P(x)**	**xP(x)**	**x - μ**	**$(x - \mu)^2$**	**$(x - \mu)^2$ P(x)**
10	.17				
15	.08				
20	.16				
25	.39				
30	.20				
		Σ =			Σ =

3) A commuter must pass through five traffic lights on her way to work, and will have to stop at each one that is red. She estimates the probability model for the number of red lights she hits as shown below.

x = # of red lights	0	1	2	3	4	5
P(x)	.05	.25	.35	.15	.15	.05

a) Construct a table to display the data use the columns from above.

b) How many red lights should she expect to hit each day.

c) What's the standard deviation?

1) The head nurse on the third floor of a community hospital is interested in the number of nighttime room calls requiring a nurse. For a random sample of nights (9:00pm to 6:00am), the following information was obtained where x = the number of calls requiring a nurse and f = the frequency with which this many calls occurred.

x	36	37	38	39	40	41	42	43	44
f	6	10	11	20	26	32	34	28	25

Find:

a) the expected value of the distribution

b) the standard deviation of the distribution

2) Commercial aircraft in the United States are aging. Aviation Data Services maintains records on the U.S. fleet of 3,946 commercial aircraft. Their records show the age distribution for the aircraft as of July 1989 (rounded to the nearest year). The information is given in the table below.

Age (in years)	**# of Aircraft**
0 to 4	**891**
5 to 9	**680**
10 to 14	**578**
15 to 19	**556**
20 and older	**1241**

Find:
a) the expected value of the distribution
b) the standard deviation of the distribution

5.1 Homework

1) Which of the following are continuous variables, and which are discrete?

(a) Number of traffic fatalities per year in the state of Florida

(b) Distance a golf ball travels after being hit with a driver

(c) Time required to drive from home to college on any given day

(d) Number of ships in Pearl Harbor on any given day

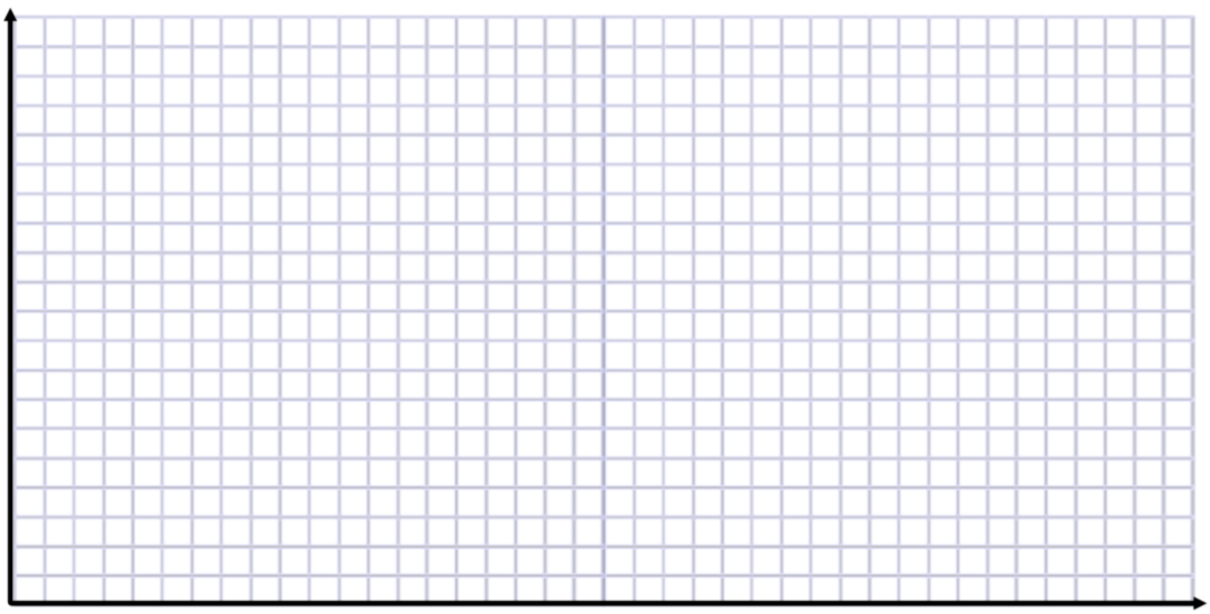
(e) Your weight before breakfast each morning

2) Consider the probability distribution of a random variable x. Is the expected value of the distribution necessarily one of the possible values of x? Explain or give an example.

3) In the following table, income units are in thousands of dollars, and each interval goes up to but does not include the given high value. The midpoints are given to the nearest thousand dollars.

Income range	5–15	15–25	25–35	35–45	45–55	55 or more
Midpoint x	10	20	30	40	50	60
Percent of super shoppers	21%	14%	22%	15%	20%	8%

(a) Use a histogram to graph the probability distribution of part (a).



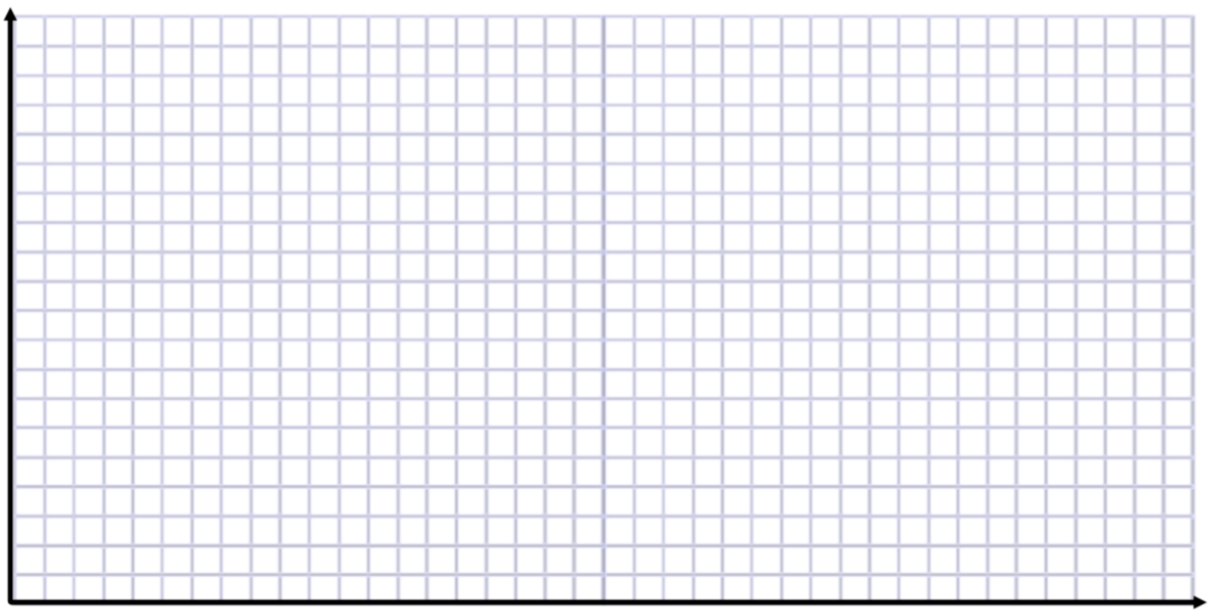
(b) Compute the expected income, and standard deviation of a super shopper.

x	$P(x)$	xP(x)	$x-\mu$	$(x-\mu)^2$	$(x-\mu)^2P(x)$
10					
20					
30					
40					
50					
60					
		$\Sigma xP(x) =$			$\Sigma(x-\mu)^2P(x) =$

4) What was the age distribution of nurses in Great Britain at the time of Florence Nightingale? Thanks to Florence Nightingale and the British census of 1851, we have the following information (based on data from the classic text *Notes on Nursing*, by Florence Nightingale). *Note:* In 1851 there were 25,466 nurses in Great Britain. Furthermore, Nightingale made a strict distinction between nurses and domestic servants.

Age range (yr)	20–29	30–39	40–49	50–59	60–69	70–79	80+
Midpoint x	24.5	34.5	44.5	54.5	64.5	74.5	84.5
Percent of nurses	5.7%	9.7%	19.5%	29.2%	25.0%	9.1%	1.8%

(a) Use a histogram to graph the probability distribution of part (a).



(b) Find the probability that a British nurse selected at random in 1851 would be 60 years of age or older.

(c) Compute the expected age and standard deviation of a British nurse contemporary to Florence Nightingale.

x	$P(x)$	xP(x)	$x-\mu$	$(x-\mu)^2$	$(x-\mu)^2 P(x)$
24.5					
34.5					
44.5					
54.5					
64.5					
74.5					
84.5					
		$\Sigma xP(x) =$			$\Sigma(x-\mu)^2 P(x) =$

5.2 Binomial Experiments

Jacob Bernoulli – studied binomial experiments in the late 1600's.

Binomial Experiments

i. There are a ________________ number of trials. We denote this number by the letter n.

i. The n trials are __________________ and repeated under identical conditions.

ii. Each trial has only ____________ outcomes: success, denoted by S, and failure, denoted by F.

iii. For each individual trial, the probability of success is the same. We denote the probability of _______________________ and that of _______________________. Since each trial results in success or failure, $p + q = 1$ and $q = 1 - p$.

iv. The central problem of a binomial experiment is to find the probability of ___________________________________.

Independence Criterion

Independence must also be satisfied in a binomial experiment. This means that the outcome of a trial ____________ affect the outcome of any other trial. Anytime we make selections from a population ___________________ replacement, we do not have independent trials.

Let's analyze the following binomial experiment to determine p, q, n, and r:

According to the *Textbook of Medical Physiology*, 5th Edition, by Arthur Guyton, 9% of the population has blood type B. Suppose we choose 18 people at random from the population and test the blood type of each. What is the probability that three of these people have blood type B? (*Note:* Independence is approximated because 18 people is an extremely small sample with respect to the entire population.)

(a) In this experiment, we are observing whether or not a person has type B blood. We will say we have a success if the person has type B blood. What is failure?

(b) The probability of success is 0.09, since 9% of the population has type B blood. What is the probability of failure, q?

(c) In this experiment, there are n = _______ trials.

(d) We wish to compute the probability of 3 successes out of 18 trials. In this case, r = _______.

Examples:

The registrar of a college noted that for many years the withdrawal rate from an introductory chemistry course has been 35% each term. If 80 students register for the course, what is the probability that 55 will complete it?

a) Each of the 80 students enrolled in the course can make the decision to withdraw or complete the course. The decision of each student can be thought of as a trial. How many trials are there?

n = ______________

b) In this problem we will assume that the decision one student makes about withdrawing from the course does not affect the decision of any other student. Under this assumption are the trials independent?

c) A trial consists of a student deciding to withdraw or complete the chemistry course. How many possible outcomes are there to each trial?

d) Let's call completing the course a success. Then, withdrawing is a failure. The probability of failure for each trial is q = .35. Find the probability of success (p) for each trial.

e) In this problem we want to know the probability of r = _________ successes out of n = _________ trials.

We want to determine if the following experiment is binomial:

A random sample of 30 men between the ages of 20 and 35 is taken from the population of Teliville. Each man is asked to name his favorite TV program.

(a) The response of each of the 30 men is a trial, so there are n = ________ trials.

(b) How many outcomes are possible on each trial? Can this be a binomial experiment?

5.2 Homework

1) What does the random variable for a binomial experiment of *n* trials measure?

2) In a binomial experiment, is it possible for the probability of success to change from one trial to the next? Explain.

3) A fair quarter is flipped eight times. If we count heads as a success, what is the probability of finding 3 successes? For this question state n, p, q, and r.

4) Richard has just been given a 10-question multiple-choice quiz in his history class. Each question has five answers, of which only one is correct. Since Richard has not attended class recently, he doesn't know any of the answers. Is this a binomial distribution? Why or why not?

5.3 The Binomial Distribution

The main part of a binomial experiment is to find the probability of r successes out of n trials. (Think of a multiple choice exam that you didn't study for. What are your possible outcomes?)

Formula for the binomial probability distribution

$$P(r) = \frac{n!}{r!(n-r)!} p^r q^{n-r} = C_{n,r} p^r q^{n-r}$$

where n = number of trials

p = probability of success on each trial

$q = 1 - p$ = probability of failure on each trial

r = random variable representing the number of successes out of n trials $(0 \le r \le n)$

$!$ = factorial notation. Recall from Section 4.3 that the factorial symbol $n!$ designates the product of all the integers between 1 and n. For instance, $4! = 4 \cdot 3 \cdot 2 \cdot 1 = 24$. Special cases are $1! = 1$ and $0! = 1$.

$C_{n,r} = \frac{n!}{r!(n-r)!}$ is the binomial coefficient. Table 2 of Appendix II gives values of $C_{n,r}$ for select n and r. Many calculators have a key designated nCr that gives the value of $C_{n,r}$ directly.

Maria is doing a study on the issue of the quarter system versus the semester system. To obtain faculty input, she mails out questionnaires to the faculty. The probability that a faculty member chosen at random returns the completed questionnaire is 0.65. Five faculty members chosen at random from the foreign language department are sent questionnaires.

a) Compute the probability that exactly two completed questionnaires are returned.

b) Compute the probability that all five are returned.

Video games are popular, and Xbox is one of the most popular. In fact, their market researchers claim that in the near future 22% of U.S. households will have the Xbox video game system. Based on this projection, what is the probability that for a random sample of 12 households, exactly 5 will have Xbox?

The calculator can do this formula for you; just know what values to put in. Under the 2^{nd} DISTR menu you will find the commands for finding Binomial probabilities.

Select **binompdf**, then after the parenthesis put in n, p, r to get the answer.

A biologist is studying a new hybrid tomato. It is known that the seeds of this hybrid tomato have probability 0.70 of germinating. The biologist plants 10 seeds.

a) What is the probability that **exactly** 8 seeds will germinate?

b) What is the probability that **at least** 8 seeds will germinate?

A service company in California is in the business of finding addresses of long lost friends. It claims to have a 70% success rate. Suppose you have the names of 6 friends for whom you have no address and decide to use this company to find them.
a) What is the probability that you find all of the lost friends?

b) What is the probability that you do not find any of the lost friends?

c) What is the probability that you find half of the lost friends?

d) What is the probability that you find at least 4 of the friends?

Three quarters of all trees planted by a certain landscaping company survive. This company recently planted 11 trees.

a) What is the probability that 7 trees survive?

b) What is the probability that no more than 5 trees die?

c) What is the probability that 3 trees die?

d) What is the probability that all of the trees survive?

e) What is the probability that less than half of the trees survive?

5.3 Homework

1) A fair quarter is flipped three times. For each of the following probabilities, use the calculator to compute the probability.

(a) Find the probability of getting exactly three heads.

(b) Find the probability of getting exactly two heads.

(c) Find the probability of getting two or more heads.

(d) Find the probability of getting exactly three tails.

2) Richard has just been given a 10-question multiple-choice quiz in his history class. Each question has five answers, of which only one is correct. Since Richard has not attended class recently, he doesn't know any of the answers. Assuming that Richard guesses on all 10 questions; find the indicated probabilities.

(a) What is the probability that he will answer all questions correctly?

(b) What is the probability that he will answer all questions incorrectly?

(c) What is the probability that Richard will answer at least half the questions correctly?

3) A research team at Cornell University conducted a study showing that approximately 10% of all businessmen who wear ties wear them so tightly that they actually reduce blood flow to the brain, diminishing cerebral functions. At a board meeting of 20 businessmen, all of whom wear ties, what is the probability that:

(a) at most two ties are too tight?

(b) no tie is too tight?

4) Trevor is interested in purchasing the local hardware/ sporting goods store in the small town of Dove Creek, Montana. After examining accounting records for the past several years, he found that the store has been grossing over $850 per day about 60% of the business days it is open. Estimate the probability that the store will gross over $850

(a) at least 3 out of 5 business days.

(b) fewer than 5 out of 10 business days.

5) Are your finances, buying habits, medical records, and phone calls really private? A real concern for many adults is that computers and the Internet are reducing privacy. A survey conducted by Peter D. Hart Research Associates for the Shell Poll was reported in *USA Today.* According to the survey, 37% of adults are concerned that employers are monitoring phone calls. Use the binomial distribution to calculate the probability that

(a) out of five adults, none are concerned that employers are monitoring phone calls.

(b) out of five adults, all are concerned that employers are monitoring phone calls.

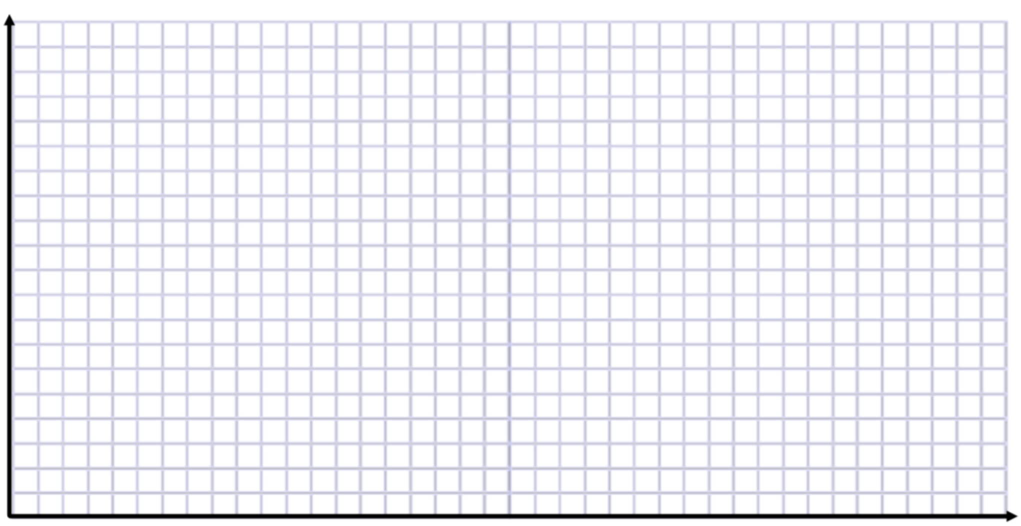
(c) out of five adults, exactly three are concerned that employers are monitoring phone calls.

5.4 Additional Properties of the Binomial Distribution

The **graph of the binomial distribution** is in the form of a histogram. The values of r or p (successes) are placed along the horizontal axis and the values of P(r) on the vertical axis. The binomial distribution is a discrete probability distribution because r can assume only whole number values such as 0, 1, 2, 3, and so on. Each bar must be only 1 unit wide because the sum of the areas of the bars must be 1.

Example:

A waiter at the Green Spot Restaurant has learned from long experience that the probability that a lone diner will leave a tip is only 0.7. During one lunch hour he serves six people who are dining by themselves. Make a graph of the binomial probability distribution which shows the probabilities that 0,1,2,3,4,5, or all 6 lone diners leave tips.



Mean and Standard Deviation of Binomial Probability Distributions
Two features that help describe the graph of any distribution are the _______________________ of the distribution and the ________________ of the distribution about the balance point.

Balance Point - _________________ of the distribution.
- The mean is the _______________________ of the number of successes.

Measure of Spread - most commonly used.
- ____

For the binomial distribution there are 2 formulas we can use to compute the mean (μ) and the standard deviation (σ).

HOW TO COMPUTE μ AND σ FOR A BINOMIAL DISTRIBUTION

$\mu = np$ is the **expected number of successes** for the random variable r

$\sigma = \sqrt{npq}$ is the **standard deviation** for the random variable r

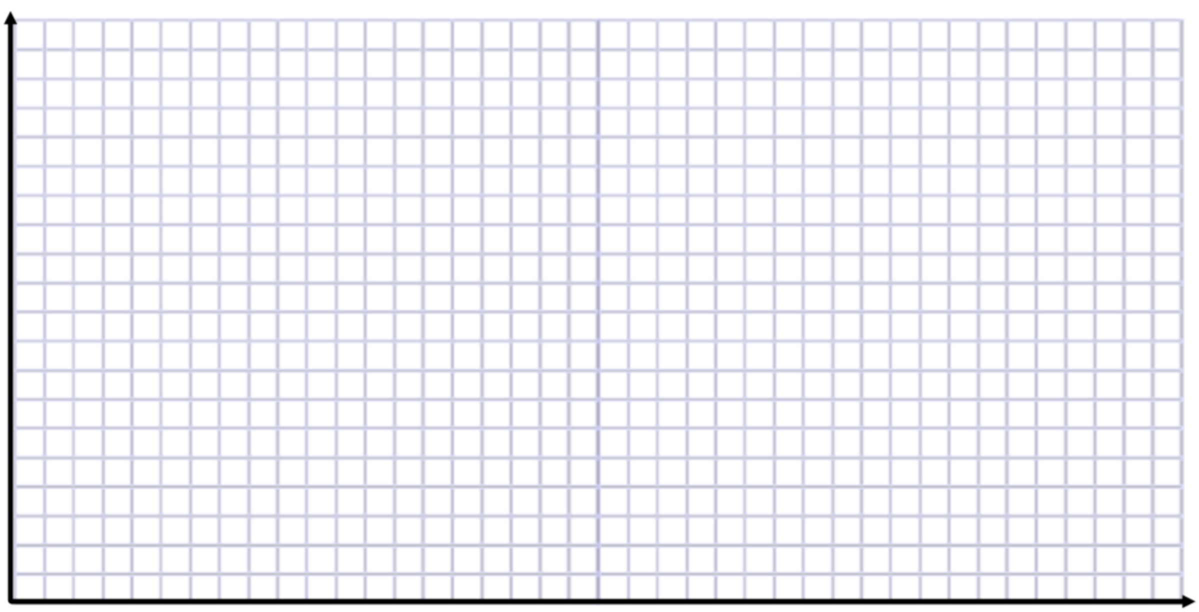
Where n is the number of trials, p is the probability of success, and q is the probability of failure (q = 1 - p).

Examples:

1) The probability that a restaurant patron will request seating in the nonsmoking section is 0.65. A random sample of 5 people calls to make reservations. Let r be the number who request seating in the nonsmoking section.

a) Find P(r) for r = 0, 1, 2, 3, 4, 5.

b) Make a histogram of the r probability distribution?



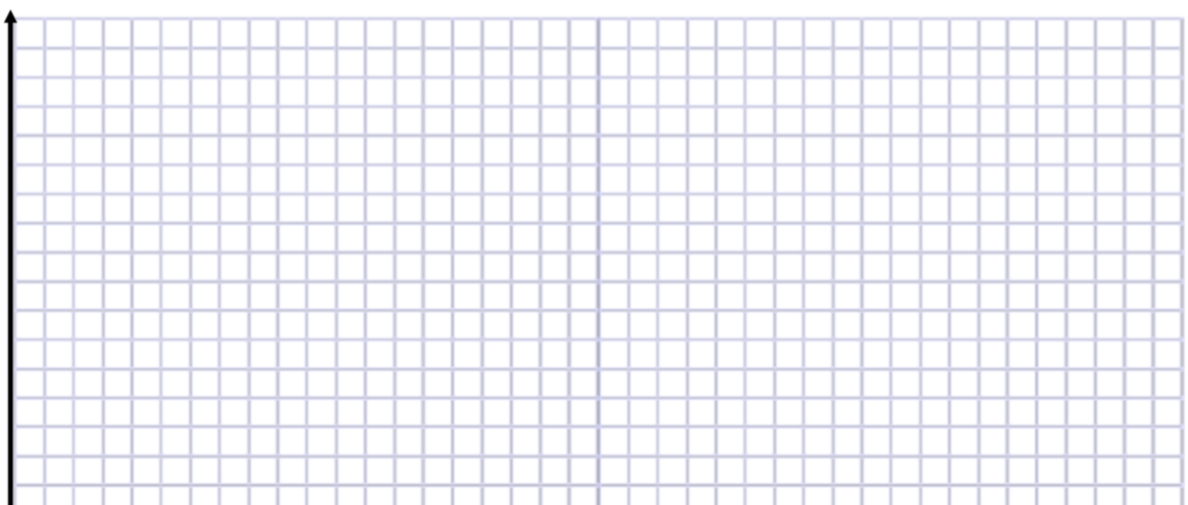
c) What is the expected number of people out of the 5 who will request a nonsmoking section?

d) What is the standard deviation of the r probability distribution?

2) Long-term history has shown that 65% of all elected offices in the Gunnison County, Colorado have been won by Republican candidates. This year there are 5 public offices up for election. Use the above estimate for the probability of a Republican being elected to office and let r be the number of public offices won by Republicans.

a) Find P(r) for r = 0, 1, 2, 3, 4, 5.

b) Make a histogram for the r probability distribution.



c) What is the expected number of Republicans who will win office in the coming election?

d) What is the standard deviation of the r distribution?

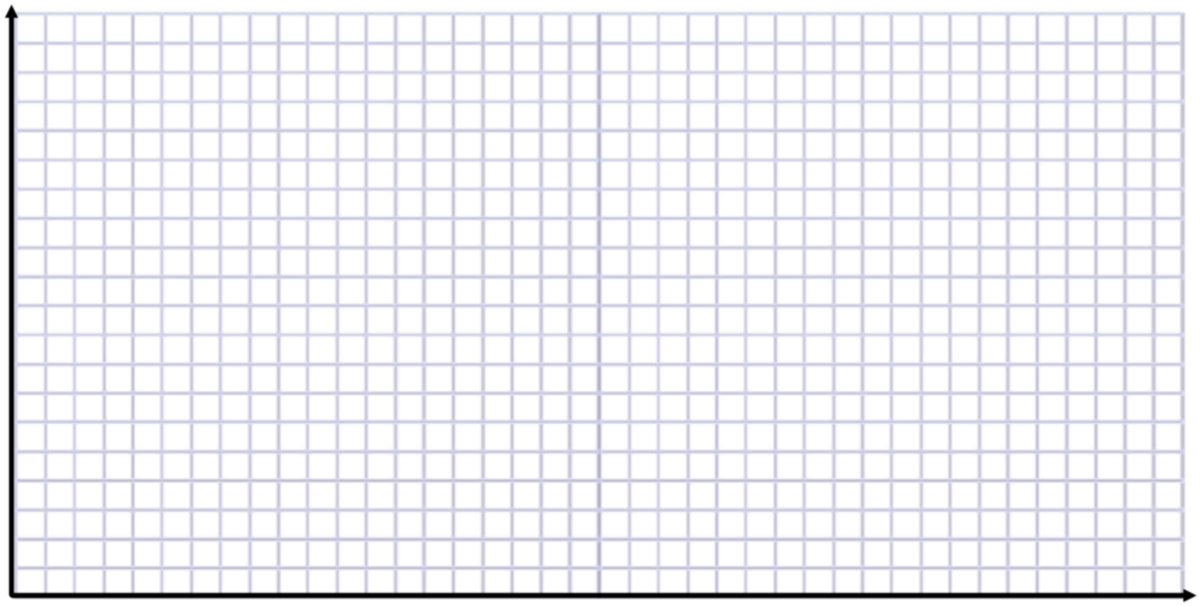
e) Find the probability that three or more Republicans will win office.

f) Find the probability that two or fewer Republicans will win office.

3) Jim is an automobile salesman at Courtesy Cars, Inc. He has a history of making a sale for about 15% of all the customers to whom he shows automobiles and goes out on test drives with eight customers. Use the above estimate for the probability of a single sale and let r be the number of sales on a day when Jim has eight customers.

a) Find P(r) for r = 0, 1, 2, 3, 4, 5, 6, 7, 8.

b) Make a histogram for the r probability distribution.



c) What is the expected number of cars Jim will sell if he has eight customers?

d) What is the standard deviation of the r distribution?

e) What is the probability that Jim will sell at least one car on a day when he has eight customers?

f) What is the probability that Jim will sell two or fewer cars on a day when he has eight customers?

5.4 Homework

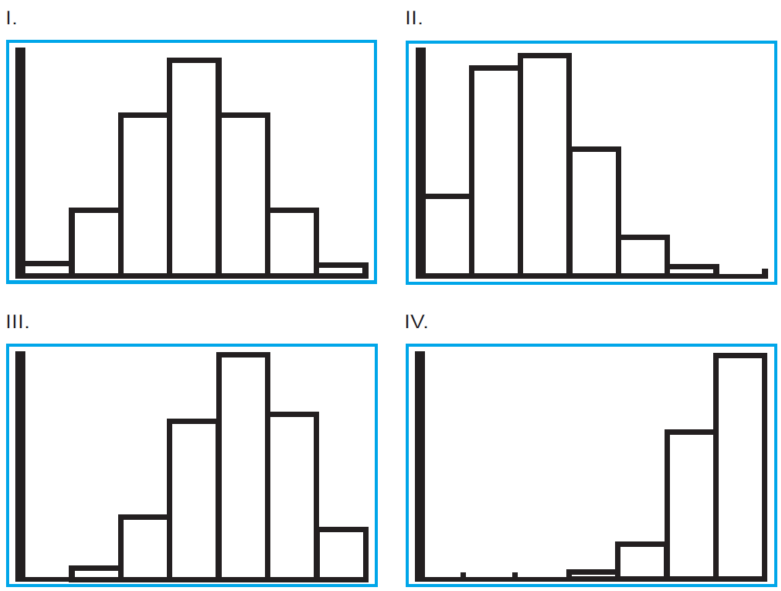
1) The figure below shows histograms of several binomial distributions with $n = 6$ trials. Match the given probability of success with the best graph.

(a) $p = 0.30$ goes with graph:

(b) $p = 0.50$ goes with graph:

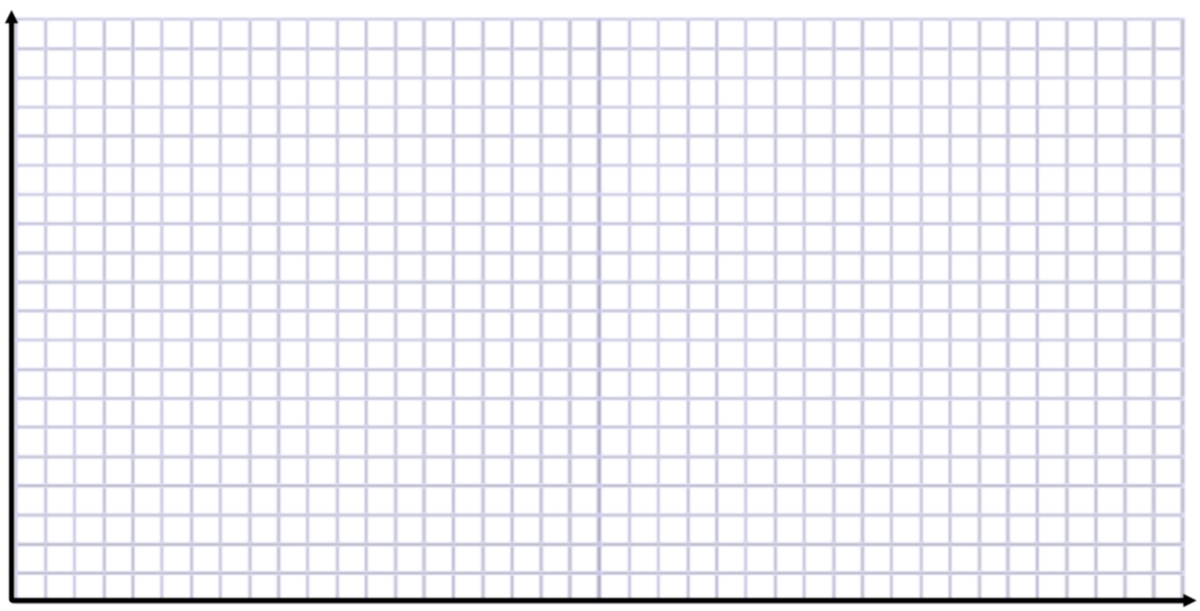
(c) $p = 0.65$ goes with graph:

(d) $p = 0.90$ goes with graph:



2) The quality-control inspector of a production plant will reject a batch of syringes if two or more defective syringes are found in a random sample of eight syringes taken from the batch. Suppose the batch contains 1% defective syringes.

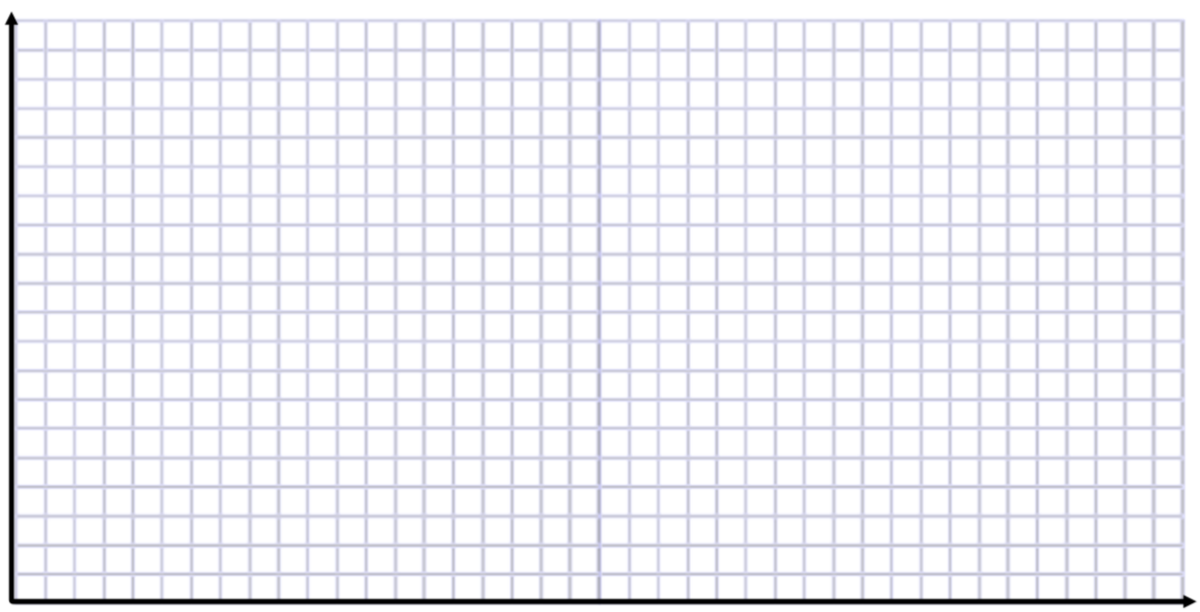
(a) Make a histogram showing the probabilities of $r = 0, 1, 2, 3, 4, 5, 6, 7,$ and 8 defective syringes in a random sample of eight syringes.



(b) Find μ and σ.

3) The Mountain States Office of State Farm Insurance Company reports that approximately 85% of all automobile damage liability claims were made by people less than 25 years of age. A random sample of five automobile insurance liability claims is under study.

(a) Make a histogram showing the probability that $r = 0$ to 5 claims are made by people under 25 years of age.



(b) Find the mean and standard deviation of this probability distribution. For samples of size 5, what is the expected number of claims made by people less than 25 years of age?

5.5 The Geometric and Poisson Probability Distributions

When we have an experiment where we repeat binomial trials we get our ______________ success and then we stop, we use n as the number of the trial on which we get our first success. That means now that n is ____________________________________. In fact, n could be any of the numbers 1, 2, 3… etc. When looking for the first trial where a success will come we use the ___________________________________.

Geometric probability distribution

$$P(n) = p(1 - p)^{n-1}$$

where n is the number of the trial on which the *first success* occurs ($n = 1, 2, 3, \ldots$) and p is the probability of success on each trial. *Note:* p must be the same for each trial.

Example:

Use the values of n = 1,2,3,4, and 5, and p = .65 to find the probability of success on each trial.

1) People with O-negative blood are called "universal donors" because only O-negative blood can be given to anyone else, regardless of the recipient's blood type. Only about 6% of people have this type of blood. If donors line up at random for a blood drive, find the probability that you will get a person with O-negative before the 7^{th} trial.

We can also use the calculator to help us quickly solve for the probability. Use the 2^{nd} VARS function and select (geometpdf). This stands for geometric probability density function. Then in the parenthesis just place the probability, followed by a comma, followed by the n which you are looking for

2) A certain factory uses robots to find malfunctions in automobiles. The robots are only successful .78 of the time. If they do not locate the malfunction then they just try again. What is the probability that the robot's first success will be on attempts n = 1, 2, 3, or 4?

3) On average only 4% of people have type AB blood.
a) What is the probability that there is a type AB blood donor **among** the first 5 people checked?

b) What is the probability that the first type AB blood donor will be found **among** the first 6 people?

c) What's the probability that we won't find a Type AB blood donor **before** the 7th person?

4) About 8% of males are colorblind. A researcher needs some colorblind subjects for an experiment and begins checking potentials subjects.

a) What's the probability that she won't find anyone colorblind **among** the first 4 men she checks?

b) What's the probability that the first colorblind man found will **be** the sixth person checked?

c) What's the probability that she finds someone who is colorblind **before** checking the tenth man?

Poisson Probability Distributions:

If in a binomial experiment the probability of success (p) gets smaller and smaller as the number of trials (n) gets larger, we have ourselves a ______________________________.

This distribution was founded by Simeon Denis Poisson (1781 – 1840). He was a French mathematician who studied probabilities of rare events that occur infrequently in space, time, volume, and so forth. This distribution applies to accident rates, arrival times, malfunction rates, incidents of bacteria in the air, smoke alarms, and many other areas of everyday life.

As with a binomial distribution, we can assume only ______ outcomes, a particular event occurs (success) or does not occur (failure) during the ________________________________ or space. These events need to be ___________________so that the one success does not change the probability of another success during the specified interval. We are interested in computing the probability of r successes in the given time period, space, volume, or specified interval.

Poisson distribution

Let λ (Greek letter lambda) be the mean number of successes over time, volume, area, and so forth. Let r be the number of successes ($r = 0, 1, 2, 3, \ldots$) in a corresponding interval of time, volume, area, and so forth. Then the probability of r successes in the interval is

$$P(r) = \frac{e^{-\lambda}\lambda^{r}}{r!}$$

where e is approximately equal to 2.7183.

1) Suppose the average number of phone calls received by a business is 4 in a 15-minute period. The business wants to determine the probability that no calls will be received during a 15-minute period.

a) What is the value of λ?

b) Find the probability that no calls will be received during a 15-minute period.

c) Find the probability that 3 calls will be received during a 15-minute period.

d) Find the probability that 6 calls will be received during a 15-minute period.

e) Find the probability that 2 calls will be received during a 30-minute period. (Hint: Does the value of λ change???)

f) Find the probability that 3 calls will be received during a 30-minute period.

g) Find the probability that 6 calls will be received during a 45-minute period.

h) Find the probability that 7 calls will be received during a 1-hour period.

We can also use the calculator to help us quickly solve for the probability. Use the 2^{nd} VARS function and select (poissonpdf). In parenthesis place your λ (lambda) value followed by your r value.

2) Sarah is a court reporter. She makes an average of 0.2 errors per page.
a) What is the probability that she will make one or fewer errors on a single page?

b) What is the probability that she will type three successive pages with no errors?

c) What is the probability that she will type 5 successive pages with no errors?

d) What is the probability that she will type 4 pages with just 1 error?

3) A loom which produces plaid wool fabric is known to produce, on the average, one noticeable flaw per 20 yards of fabric.
a) What is the probability that there will be exactly two flaws in a twenty-yard piece of the wool?

b) What is the probability that there will be one flaw in a forty-yard piece of wool?

c) What is the probability that there will be 5 flaws in a 120-yard piece of wool?

4) Mammon Savings and Loan is concerned about errors which may occur when transactions are entered into the computer. A study has revealed that a random sample of 5000 transactions revealed 500 errors or .1 errors per transaction.
a) What is the probability that there will be no errors in a transaction?

b) What is the probability that there will be one or more errors in a transaction?

c) What is the probability that there will be two errors in a transaction?

d) What is the probability that there will be three errors in a transaction?

5) A rare blood condition is found in only 2% of the population.
a) What is the mean number of people in a random sample of 500 who would have the blood condition?

b) Find the probability that no one in a sample of 500 people would have the condition.

c) Find the probability that 2 in a sample of 500 people will have the condition.

d) Find the probability that 4 in a sample of 500 people will have the condition.

5.5 Homework

1) For a binomial experiment, what probability distribution is used to find the probability that the *first* success will occur on a specified trial?

2) When using the Poisson distribution, which parameter of the distribution is used in probability computations? What is the symbol used for this parameter?

3) Bob is a recent law school graduate who intends to take the state bar exam. According to the National Conference on Bar Examiners, about 57% of all people who take the state bar exam pass. Let $n = 1, 2, 3 \ldots$ represent the number of times a person takes the bar exam until the *first* pass.

(a) What is the probability that Bob first passes the bar exam on the second try ($n = 2$)?

(b) What is the probability that Bob needs three attempts to pass the bar exam?

(c) What is the probability that Bob needs more than three attempts to pass the bar exam?

4) On the leeward side of the island of Oahu, in the small village of Nanakuli, about 80% of the residents are of Hawaiian. Let $n = 1, 2, 3 \ldots$ represent the number of people you must meet until you encounter the *first* person of Hawaiian ancestry in the village of Nanakuli.

(a) Compute the probabilities that $n = 1$, $n = 2$, and $n = 3$.

(b) Compute the probability that $n \geq 4$.

5) Approximately 3.6% of all (untreated) Jonathan apples had bitter pit in a study conducted by the botanists Ratkowsky and. (Bitter pit is a disease of apples resulting in a soggy core, which can be caused either by overwatering the apple tree or by a calcium deficiency in the soil.) Let n be a random variable that represents the first Jonathan apple chosen at random that has bitter pit.

(a) Find the probabilities that $n = 3$, $n = 5$, and $n = 12$.

(b) Find the probability that $n \geq 5$.

6) At Burnt Mesa Pueblo, in one of the archaeological excavation sites, the artifact density (number of prehistoric artifacts per 10 liters of sediment) was 1.5. Suppose you are going to dig up and examine 50 liters of sediment at this site. Let $r = 0, 1, 2, 3 \ldots$ be a random variable that represents the number of prehistoric artifacts found in your 50 liters of sediment.

(a) Explain why the Poisson distribution would be a good choice for the probability distribution of *r*. What is λ?

(b) Compute the probabilities that in your 50 liters of sediment you will find two prehistoric artifacts, three prehistoric artifacts, and four prehistoric artifacts.

(c) Find the probability that you will find three or more prehistoric artifacts in the 50 liters of sediment.

(d) Find the probability that you will find fewer than three prehistoric artifacts in the 50 liters of sediment.

7) In his doctoral thesis, L. A. Beckel (University of Minnesota, 1982) studied the social behavior of river otters during the mating season. An important role in the bonding process of river otters is very short periods of social grooming. After extensive observations, Dr. Beckel found that one group of river otters under study had a frequency of initiating grooming of approximately 1.7 for each 10 minutes. Suppose that you are observing river otters for 30 minutes. Let $r = 0, 1, 2 \ldots$ be a random variable that represents the number of times (in a 30-minute interval) one otter initiates social grooming of another.

(a) Explain why the Poisson distribution would be a good choice for the probability distribution of r. What is λ?

(b) Find the probabilities that in your 30 minutes of observation, one otter will initiate social grooming four times, five times, and six times.

(c) Find the probability that one otter will initiate social grooming four or more times during the 30-minute observation period.

(d) Find the probability that one otter will initiate social grooming less than four times during the 30-minute observation period.

8) The *Denver Post* reported that, on average, a large shopping center had an incident of shoplifting caught by security once every three hours. The shopping center is open from 10 A.M. to 9 P.M. (11 hours). Let r be the number of shoplifting incidents caught by security in an 11-hour period during which the center is open.

(a) Explain why the Poisson probability distribution would be a good choice for the random variable r. What is λ?

(b) What is the probability that from 10 A.M. to 9 P.M. there will be at least one shoplifting incident caught by security?

(c) What is the probability that from 10 A.M. to 9 P.M. there will be at least three shoplifting incidents caught by security?

(d) What is the probability that from 10 A.M. to 9 P.M. there will be no shoplifting incidents caught by security?